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C.Emre Alper

Aras Akdemir

Kazim Kazimov

ARAŞTIRMA RAPORU

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ESTIMATING THE TERM STRUCTURE OF GOVERNMENT SECURITIES IN TURKEY *

C. Emre Alper † Aras Akdemir Kazim Kazimov

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Abstract

This paper uses statistical techniques to estimate monthly yield curves in Turkey using secondary government securities data from 1992 to 2004. We use both spline based method of McCulloch and parsimonious model of Nelson-Siegel to estimate monthly yield curves. Instead of using end-of-month values, which is a common practice in the previous empirical studies, we construct the data set by calculating monthly volume weighted average of price and maturity. We use both in-sample and out-of-sample analysis to compare McCulloch and Nelson-Siegel methods. We find evidence that that McCulloch method has superior in-sample properties, whereas Nelson-Siegel method has superior out-of-sample properties.

JEL classification: E43, E47, C52

Keywords: Term premium, Yield curve estimation, McCulloch method, Nelson-Siegel model

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†Corresponding author. Professor, Department of Economics, Boğaziçi University, Bebek 34342, Istanbul, Turkey. E-mail: alper@boun.edu.tr

1 Introduction

The term structure theory of interest rate determination has been subjected to major advances in the last two decades. The progress can be broadly categorized into two: first category emphasized equilibrium models whereas the second category uses statistical techniques to measure the term structure. The equilibrium models focus on modelling the dynamics of the short rate. These models are essentially affine models, however one may also observe quadratic models in the recent literature. Basically, these models relate the short rate to some underlying state variables and model these state variables as stochastic processes. Seminal papers of the equilibrium models approach include Vasicek (1977), Cox, Ingersoll and Ross (1985), and Duffie and Kan (1996). Statistical method of measuring term structure, on the other hand, focus on obtaining a continuous yield curve from cross-sectional term structure data using curve fitting techniques. These methods can also be broadly divided into two: the former using splines and the latter using parsimonious functional forms to fit the term structure.

Spline-based models owe to the seminal paper of McCulloch (1971). McCulloch used quadratic splines to fit the term structure data. Vasicek and Fong (1981) introduced exponential splines, and Shea (1984) used B-splines for this purpose. Parsimonious models, on the other hand, specify a functional form for either the spot rate, the discount rate or implied forward rate. Chambers *et al.* (1984) proposed a simple polynomial for the spot rate. On the other hand, Nelson-Siegel (1987) used an exponential function for the implied forward rate and analytically solved for the spot rate.

This paper uses statistical techniques to estimate monthly yield curves in Turkey using secondary market government securities data from 1992 to 2004. The estimation is conducted using two different methods: the spline-based model of McCulloch (1975) and the parsimonious model of Nelson-Siegel (1987). Relative performances of models are compared using in-sample goodness of fit and out-of-sample forecasting ability following Bliss (1997).

Instead of using end-of-month values, which is a common practice in the previous empirical studies, we constructed the data set by calculating monthly volume weighted average of price and maturity. Before estimating the monthly yield curves stylized facts about the Turkish Secondary Government Securities Market (Secondary Market, henceforth) is provided. Descriptive statistics concerning Secondary Market concentration, impact of withholding tax on yields, maturity and yield comparison of the Primary and Secondary Markets are provided.

We proceed as follows. Section 2 provides basic definitions of the term structure theory. Section 3 presents the methodology used in estimations. Section 4 describes the Secondary Market and the data used in estimations. The results of the estimations and comparison of the relative performance of the methods are provided in section 5. Section 6 concludes and provides direction for further research.

2 An Overture to Terminology

This section will introduce the terminology that will be used for modelling the term structure in the following section. In order to keep the algebra uncluttered we exclude coupon payments from the definitions noting that on average more than 95% of the Secondary Government Securities Market in Turkey comprise of treasury bills and zero coupon government bonds, during the period 1992 -2004 (Table 1).

2.1 Yield to Maturity

We first introduce the concept of *yield*, technically termed as *Yield to Maturity.* Yield is the interest rate at which the present value of the cash flow at maturity is equal to its current price. In literature two kinds of yields are widely used. These are simple yield and continuously compounded yield. We denote simple yield by R and continuously compounded yield by r.

Definition 1 *For a zero-coupon bond with face value 1 TL maturing in m periods, if the current price is P, simple yield R satisfies*

$$P = \frac{1}{(1+R)^m}$$

Definition 2 *Continuously compounded yield is the log of one plus the simple yield. For a zero-coupon bond with face value 1 TL maturing in m periods, continuously compounded yield, r, satisfies*

$$P = e^{-m \cdot r}$$

Note that $(1+R)$ is also called *gross yield* in literature. It is more convenient to work with continuously compounded yields, thus we will use continuously compounded yields from now on.

2.2 Spot rate, Implied Forward rate and discount function

After defining yield we next introduce the spot rate, implied forward rate, and the discount rates.

The yields defined in the previous section are also referred as spot rates. An analysis based on spot rates (yields), for different maturities, gives information about the term structure of interest rates. When we plot the spot rates against maturities, we get the *spot yield curve*, $\eta(t)$.

We next turn to the issue of defining the forward rate. We first differentiate implied forward rate from market forward rate. Implied forward rate is a derived theoretically from spot rates. Market forward rate, on the other hand, is the actual rate that is realized in a forward or a future contract.

Consider the following simple example to clarify the concept of implied forward rate. Suppose that r_1 and r_2 are spot rates for one and two year bonds respectively. Also suppose that we have TL 1 to invest for two years. We consider the following two alternatives: investing in the two year bond and receiving e^{2r_2}, annually; investing in one year bond today and receiving e^{r_1} at the end of the year and reinvesting this amount in the one year bond having yield r_1^2, which is the year two spot yield for the one year bond. No arbitrage principle suggests that r_1^2 must satisfy $e^{2 \cdot r_2} = e^{r_1} \cdot e^{r_1^2}$. The yield satisfying this condition is also referred as the implied forward of a one year bond belonging to the next period, ϕ_2. Since r_1 and r_2 are available, one can easily calculate ϕ_2. When we extend this argument to an n-period case, the formula becomes

$$e^{-n \cdot r_2} = e^{-r_1} \cdot e^{-\phi_2} \cdot e^{-\phi_3} \cdots e^{-\phi_n}$$

By solving this equation recursively, one can calculate implied forward rates for all n periods, given the spot rates. An important result is that spot rate is the average of forward rates. Taking the periods infinitesimally closer we obtain the *instantaneous implied forward rate*, $\rho(t)$. Plotting the instantaneous implied forward rates against maturities gives us the *implied forward rate curve.*

The present value of the future cash flow to be occurred in m periods can be computed by multiplying cash flow, FV, by the discount function, $\delta(m)$:

$$PV(FV) = \delta(m) \cdot FV$$

We note that $\delta(m)$ is continuous function of time. For example, the discount function used in evaluating continuously compounded yields is

$$\delta(m) = e^{-m \cdot r}$$

Plotting discount function against time gives the discount curve. It is also important to note that by construction, discount function is between zero and one, starting from one and monotonically declining to zero as m approaches to infinity.

2.3 Calculating spot rates, implied forward rates and discount functions [1]

We next consider relation between discount function, $\delta(m)$ and instantaneous implied forward rate, $\rho(m)$:

$$\rho(m) = -\delta'(m)/\delta(m)$$

where $\delta'(m)$ is the derivative of the discount function. Since spot rates are the averages of forward rates, spot rate function is

$$\eta(m) = \frac{1}{m} \int_0^m \rho(\mu) d\mu$$

[1] See Anderson(1996) for more details

Using the relation between discount function and instantaneous implied forward rate, spot rate function is $\eta(m) = \int_0^m \delta(m)'/\delta(m)d\mu$. Taking the integration yields

$$\eta(m) = \frac{-\ln(\delta(m))}{m}$$

The previous three equations show that knowledge of any one of these functions is sufficient to solve for the other two.

3 Methodology

In this section we discuss the two methods we use in the paper for fitting the yield curve. Firstly we describe the McCulloch model and then the Nelson-Siegel model in detail.

3.1 McCulloch Estimation Technique

McCulloch (1971) developed a useful technique for fitting the discount function to observations on prices of government securities with varying maturities and coupon rates. Initially he assumed a functional form for $\delta(m)$, and transformed the relation between discount function and prices into a linear one to be estimated by ordinary least squares (OLS) technique.

In our analysis, we use simplified version of McCulloch's (1971) technique, since our data contains only treasury bills and government bonds with no coupon payments. The current price of a security, P, with face value FV, maturing at m is:

$$P = FV \cdot \delta(m)$$

In order to fit a curve to a discount function by linear regression, McCulloch postulated k continuously differentiable functions, $f_i(m)$, and then expressed the discount function as a constant term plus a linear combination of these functions:

$$\delta(m) = a_0 + \sum_{j=1}^{k} a_j f_j(m)$$

Obvious properties of $\delta(m)$ are $\delta(0) = 1$, and it must be a decreasing function of time. In order to impose these properties on $\delta(m)$, he set $a_0 = 1$ and $f_j(0) = 0$,

$\forall j$. Substituting the discount function:

$$P = FV \cdot \left(1 + \sum_{j=1}^{k} a_j f_j(m)\right)$$

There are many methods for choosing a functional form for $f_j(m)$ and a value for k, none of which can be considered as superior. One simple form could be $f_j(m) = m^j$, which makes $\delta(m)$ a polynomial of degree k. McCulloch (1971) set $f_j(m)$ to be a piecewise defined quadratic polynomial, also referred to as a quadratic spline. However, since quadratic functions have discontinuous second order derivatives, discount functions calculated by this method may result in forward rate curves with "knuckles" as McCulloch himself names them. McCulloch(1975) revised continuously differentiable functions and replaced quadratic splines with cubic splines[2].

Setting $y = P - FV$ and $x_j = FV \cdot f_j(m)$, we transform the price equation as:

$$y = \sum_{j=1}^{k} a_j x_j$$

The existence of transaction costs, tax exemptions, imperfect arbitrage, rigidities etc. transforms the exact functional relationship to:

$$P_i = FV_i \cdot \delta(m_i)) + \epsilon_i$$

and so the final equation has an added stochastic disturbance term[3].

$$y_i = \sum_{j=1}^{k} a_j x_{ij} + \epsilon_i$$

In this equation the dependent variable, y, is a linear function of known variables, x_j, and unknown coefficients a_j and can be estimated by simple linear regression methods. Coefficients of x_j can then used to calculate discount function, $\delta(m)$, which in tern give functions for zero coupon yield, $\eta(m)$ and instantaneous forward $\rho(t)$.

[2]See Appendix for the functional form of cubic spline.

[3]According to findings of Bliss (1997), final equation is misspecified and should include some other factors such as tax effects

3.2 Nelson-Siegel Model

Nelson and Siegel (1987) introduced a model for yield curves which explained 96% of the variation of the yield curve across maturities using US zero-coupon government securities market data. The model's prevalence was its ability to describe variation in yield curve using only few parameters.

Motivation for Nelson-Siegel model comes from the expectations hypothesis. According to the expectations hypothesis, forward rates will behave in such a way that there is no arbitrage opportunity in the market. In other words, the theory suggests that implied forward rates are the rationally expected spot rates of the future periods. Nelson and Siegel (1987) propose that if spot rates are generated by a differential equation, then implied forward rates will be the solutions to this equation. Assuming that spot rates are generated by a second order differential equation, instantaneous implied forward rate function can be written as

$$\rho(m) = \beta_0 + \beta_1 \cdot e^{-m/\tau_1} + \beta_2 \cdot e^{-m/\tau_2}$$

where τ_1 and τ_2 are the real roots of the differential equation. However, Nelson and Siegel concluded that the model is over-parameterized and decided to use equal root solution. Hence instantaneous implied forward rate function becomes

$$\rho(m) = \beta_0 + \beta_1 \cdot e^{-m/\tau} + \beta_2 \cdot \frac{m}{\tau} \cdot e^{-m/\tau}.$$

Using $\eta(m) = 1/m \int_0^m \rho(x) \cdot d(x)$, spot rate can be written as

$$\eta(m) = \beta_0 + \beta_1 \cdot \left(\frac{1 - e^{-m/\tau}}{m/\tau} \right) - \beta_2 \cdot \left(\frac{1 - e^{-m/\tau}}{m/\tau} - e^{-m/\tau} \right)^4.$$

The spot rate curve can take the form the form of monotonic, S-shaped and humped curves, which are shapes usually associated with yield curves.

Nelson and Siegel showed that another merit of the model is the ease with which its parameters can be interpreted. Taking the limits of the spot rate curve as m approaches to zero and infinity Nelson and Siegel found that the contribution of the long term component is β_0 and the contribution of the short term component is $\beta_0 + \beta_1$. Also $-\beta_1$ can be interpreted as term premium, since it is the difference between long term and short term yields.

[4] We use the factorization of yield curve suggested by Diebold and Li(2003)

Another interpretation of the parameters of the Nelson-Siegel model is suggested by Diebold and Li (2003). According to Dielold and Li (2003), β_0, β_1 and β_2 can be interpreted as three latent factors of the yield curve. The loading on β_0 is 1 and does decay to zero in the limit; thus it can be interpreted as the yield curve level. Also, an increase in β_0 increases all yields equally. As we have noted, β_1 is the short-term yield and can be interpreted as slope of the yield curve. Note that an increase in β_1 increases short yields more than long yields, thereby changing the slope of the yield curve. Finally, β_2 can be interpreted as curvature of the yield curve. An increase in β_2 has little effect on very short and very long yields, but increase the medium term yields.

The parameters of the Nelson Siegel model are estimated by nonlinear regression using the constrained maximum likelihood procedure.[5] During estimations heteroscedasticity is corrected trough weighting each residual by its maturity, as suggested by Bliss (1997). Bliss used inverse of maturities as weights since he estimated the parameters by price errors as residuals, whereas we used yield errors as residuals.

4 The Data

We used daily weighted average prices and yields of bonds and t-bills from January 1992 trough March 2004 taken from Istanbul Stock Exchange (ISE) daily bulletins.

Primary market auctions of government securities started on May 1985 and the secondary market was initiated on June 1991. The volume of transactions of ISE Bonds and Bills Market has increased substantially since 1991 (Figure 1).

Compared to industrialized countries' fix income securities markets, Turkish Bonds and Bills Market has noticeably lower average maturity. Figure 2 shows average of maturities in primary and secondary markets weighted by volume of each individual security. Maturities in primary market range from two months to two years with average of nine months. Secondary market has even shorter maturity horizon ranging from two months to slightly more than one year. Av-

[5]Estimations are done using GAUSS. Included among constraints is the restriction $\eta(0) \geq 0$. See Appendix for gauss codes.

erage maturity in secondary market is six months. The scarcity of securities with maturities longer than a year can be explained by the instable macroeconomic environment in the post-financial account liberalization period of the 1990s characterized by high inflation, high public sector borrowing requirements and frequent financial flow reversals as well as immature financial markets.

Turkish Secondary Government Securities Market is highly concentrated (Table 2). In some months over 90% of the volume of volume of transactions is generated just by one benchmark treasury bill! The last column represents the Herfindahl-Hirschmann index for shares of securities in the total volume in the market. It is calculated as the sum of squared shares of each security traded during the month in total trade volume. The index takes on a value between zero and one, values closer to one indicate increased concentration in the government securities market.

Before estimating monthly yield curves, the raw database was refined based on three different filters. First, we excluded coupon bonds (both with fixed and floating coupon payments), floating rate notes, government bonds denominated in foreign currency, and some other special purpose bonds. Also, we excluded stripped notes since usually only floating rate notes are stripped. Although it is desirable to have fixed coupon bonds in the database in order to increase maturity horizon, we chose to exclude them since they are illiquid and we lack information about their coupon rates and coupon frequencies. The first set of exclusions can be seen under "Filter 1" column in Table 1. On the average, they constitute only 3.08% of the total monthly volume. The discrete jump in weight of exclusions after 1999 can be attributed to the increase in floating rate notes issued during the period in which Turkey embarked in a three year IMF-backed disinflation and stabilization program.

Second, we excluded treasury bills subject to withholding tax with simple yield less than minimum overnight rates between March 1997- July 1999[6]. The reason for the exclusion can be stated as follows: during the period, treasury bills subject to withholding tax having maturity less than 3 months traded substantially below the minimum overnight rates market average (Figure 4) because

[6]Withholding tax is levied on the face value. Tax rate is determined by the yield occurred in the bond auction.

of some tax advantages to corporate sector. Column “Filter 2” in Table 1 gives information about the weight of this kind of bills in market as percentage of total volume following the first set of exclusions.

Third and finally, treasury bills with maturity less than 10 days were excluded due to liquidity problems. As can be seen form column “Filter 3” in Table 1 this constitutes on average 1.19% of the total volume following exclusion sets 1 and 2.

The weight of the total exclusions can be observed from the final column in Table 1. On average 4.38% of total volume is excluded from the database, which is not significant. considering the highly concentrated structure of Turkish bond market (Table 2).

Turkish secondary government securities market is relatively shallow. The market is characterized by small number and low volume of transactions that vary from day to day within a month. Due do this reason, instead of using end-of-month observations, which is a common practice in literature, we use volume-weighted monthly average observations. For example, in order to calculate continuously compounded yields, for each security, we calculate the daily volume-weighted average of tax adjusted yields and days to maturity for the month. Next using calculated average yields and days to maturity, we obtain the price of each bond with a normalized face value of 100,000 TL.

5 Results

5.1 Estimation Results

For the Nelson-Siegel model estimated parameters turned out to be all statistically significant for each month during January 1992- March 2004[7]. In many cases positivity constraint is binding.

The yield curves estimated by Nelson-Siegel method are mostly upward sloping and humped yield curves. We also observed some inverted yield curves. Examples of monthly estimated yield curves are given in Figure 5. As noted,

[7]For each estimated parameter t-statistics are computed using White Heteroscedasticity consistent variance-covariance matrix

$-\beta_1$ can be interpreted as the term premium. According to our estimations, the estimated term premium parameter is negative in 21 months and ten of these months correspond to the period of the stabilization and the disinflation program between December 1999 and November 2000 (Figure 6).

Alternatively we calculate the term premium by subtracting the estimated continuously compounded yield for one month from that of a year, using the two methods. Figure 7 exhibits the estimated term premia by Nelson-Siegel and McCulloch methods. Significant differences between the estimated term premia can be explained by the low out-of-sample prediction ability of the McCulloch method in the long end of the yield curve[8]. We note that significant differences between the estimated term premia using the two methods often take place in months where securities with maturity one year or higher are not traded. Hence, in these months, McCulloch method overestimates the one year yield, consequently overestimating term premium. We also observe that the estimated term premia using Nelson-Siegel method is usually lower. The reason is that Nelson-Siegel method tends to overestimate the short-run yield, whereas McCulloch method tends to overestimate the long-run yield.

5.2 Comparison Techniques for Different Yield curve Estimations

Following the estimation of monthly yield curves using the two methods, we compare their relative performance. The criteria for comparing the relative performance include root mean squared yield error (RMSYE), mean absolute yield error (MAYE) and weighted mean absolute yield error (WMAYE). For each month, let ε_i denote the gap between actual and fitted yields for the i^{th} bill in the sample. The error measures are given by:

$$RMSYE = \sqrt{\frac{1}{N}\sum_{i=1}^{N} {\varepsilon_i}^2}$$

$$MAYE = \frac{1}{N}\sum_{i=1}^{N} |\varepsilon_i|$$

[8]This point will be taken up in detail in section 5.4.

$$WMAYE = \frac{1}{N}\sum_{i=1}^{N} w_i|\varepsilon_i|$$

where $w_i = D(m_i)/\sum_{i=1}^{N} D(m_i)$ and $D(m_i)$ is the duration which equals number of days to maturity in zero coupon bonds. This weighting scheme is proposed by Bliss (1997) who argues that this approach reflects the theoretical relation between bond prices and interest rates.

These error measures can be used to compare performances of methods among themselves as well as across different maturity scopes. For example, RMSYE of either of the two methods for long and short periods state the relative capability at different ends of the yield curve.

5.3 In-sample Analysis

Absolute yield error, maturity weighted absolute yield error and root absolute yield error are computed for each month. Annual averages are given in Tables 3 and 4. Table 5 prevents number of months each method outperforms the other as well as classification of RMYSE statistic across four maturity intervals. McCulloch method performs better than Nelson-Siegel according to all three criteria. McCulloch outperforms Nelson-Siegel method in 102 months out of 147 months with respect to the MAYE statistic, 91 months with respect to WMAYE statistic and 101 months with respect to RMSYE statistic.

Classification of errors between different maturity intervals shows that ability of Nelson-Siegel method in fitting the term structure data increases with maturity, whereas fitting ability of McCulloch method decreases. According to Table 5, McCulloch method outperforms Nelson-Siegel method in 101 months when the maturities are less than 180 days. However, when we consider the 180-270 days interval this number decreases to 77 and decreases to 59 when the maturities are greater than 270 days.

5.4 Out-of-sample Analysis

Very short maturities in Market creates difficulties in testing out-of-sample performance of yield curves. So use of cross-validation technique explained below is necessary to perform the test.

Out-of-sample analysis can be conducted in two ways. First, one can estimate the yield curve using observation from a sub-sample of maturities and then using the resulting estimated parameters forecast the excluded yields. The difference between actual and forecasted yield is called out-of-sample error. Even though this method is intuitive and straightforward, we chose not to use it due to insufficient number of securities traded in the database. Secondly one may use the Cross-Validation method which may be described as follows. Each month, subsamples are formed by excluding one observation at a time and then parameters are estimated with remaining observations and fitted yield of excluded variable is computed using its days to maturity. This procedure is repeated for each observation and RMSYE statistic for each month is computed. We also categorize each excluded variable according to its maturity. Four categories are formed as in the in-sample analysis.

Contrary to in-sample analysis, Nelson-Siegel method outperforms McCulloch method in all categories (Table 6). Prediction ability of McCulloch method decreases with maturity. However, Nelson-Siegel method follows a different pattern from the in-sample analysis. Nelson-Siegel method predicts much better in 90-180 and 180-270 intervals than in 0-90 and 270- intervals. In other words, relative out-of-sample performance of Nelson-Siegel is better in the middle region when compared to long and the short ends of the yield curve. Table 6 shows the number of months Nelson-Siegel outperforms McCulloch method and vice versa. Finally Tables 7 and 8 presents annual averages for out-of-sample errors' statistics.

6 Conclusion

In this paper, we describe the Turkish Secondary Government Securities Market and attempt to estimate the monthly term premium from January 1992 to March 2003. Some of the stylized facts about Turkish secondary bond market can be stated as follows. Average maturity is low compared to industrialized countries' fix income securities markets. However, average maturity has been steadily increasing excluding periods of financial crises. Secondary markets transaction volume has increased until the year 2000. It has decreased after the crises of

November 2000- February 2001 and has slowly approached its pre-crisis level in the year 2003. Maturity structure of primary and secondary market follow the same pattern. Finally Turkish secondary bond market is highly concentrated; majority of the transaction volume is generated by two or three benchmark securities.

In order to estimate the monthly term premium, we estimate monthly yield curves using McCulloch and Nelson-Siegel methods. We also compare the relative performance of the two methods by considering their in-sample fit and out-of-sample forecasting abilities. We find evidence that that McCulloch method has superior in-sample properties, whereas Nelson-Siegel method has superior out-of-sample properties. Specifically, the Nelson-Siegel method in the middle region of the yield curve fits better.

Directions for further research include estimating and interpreting the time series properties of level, slope and curvature factors of monthly yield curves. Moreover, a multi-factor affine term structure model with this factors and some additional macro state variables can be constructed to assess the effects of macro factors on the term structure of interest rates.

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APPENDIX

Cubic Spline

The functional form proposed by McCulloch (1975) for $f_j(m)$ in

$$\delta(m) = 1 + \sum_{j=1}^{k} a_j f_j(m)$$

is a piecewise defined cubic polynomial, also referred to as a cubic spline. In order to define a cubic spline polynomial, first, a set of $k-1$ knot points, d_j, should be selected so that approximately equal number of maturities exist between adjacent knots. Then, for $m < d_{j-1}$

$$f_j(m) = 0$$

for $d_{j-1} \leq m < d_j$,

$$f_j(m) = \frac{(m - d_{j-1})^3}{6(d_j - d_{j-1})}$$

for $d_j \leq m < d_{j+1}$,

$$f_j(m) = \frac{c^2}{6} + \frac{ce}{2} + \frac{e^2}{2} - \frac{e^3}{6(d_{j+1} - d_j)}$$

where $c = d_j - d_{j-1}$ and $e = m - d_j$,
for $d_{j+1} \leq m$,

$$f_j(m) = (d_{j+1} - d_{j-1}) \left[\frac{2d_{j+1} - d_j - d_{j-1}}{6} + \frac{m - d_{j+1}}{2} \right]$$

(set $d_{j-1} = d_j = 0$ when $j = 1$)

The formulae apply for $j < k$. For $j = k$,

$$f_k(m) = m$$

regardless of m.

For the value of k McCulloch (1971) proposed the nearest integer to $\sqrt{n}$, where n is the number of securities used in estimation. He claims that this formulation will give approximately the same results with other values of k, chosen to minimize the estimator of error variance, without the expensive search.

Tables

Table 1: Excluded Data Statistics (millions)

Year	Initial Volume		Filter 1	Filter 2	Filter 3	Total
1992	186,700 TL	$2,700	0.49%	0.00%	3.96%	4.44%
1993	1,153,900 TL	$10,500	0.48%	0.00%	1.99%	2.47%
1994	2,847,300 TL	$9,800	1.89%	0.00%	4.80%	6.69%
1995	4,879,600 TL	$19,800	0.48%	0.00%	0.67%	1.15%
1996	11,048,900 TL	$44,600	0.02%	0.00%	0.11%	0.13%
1997	14,992,100 TL	$58,900	1.06%	0.12%	0.10%	1.29%
1998	25,835,600 TL	$101,200	0.47%	0.99%	0.30%	1.75%
1999	36,829,100 TL	$138,100	8.68%	0.25%	0.17%	9.10%
2000	86,772,400 TL	$327,900	5.87%	0.00%	0.18%	6.05%
2001	12,898,500 TL	$43,400	5.61%	0.00%	0.91%	6.52%
2002	21,569,500 TL	$83,000	7.20%	0.00%	0.74%	7.94%
2003	36,995,400 TL	$180,400	3.32%	0.00%	0.42%	3.74%
2004	12,334,200 TL	$68,600	8.49%	0.00%	0.88%	9.36%
Average	1,825,400 TL	$7,400	3.08%	0.11%	1.19%	4.38%
Max	11,116,600 TL	$42,200	37.31%	2.37%	11.96%	37.31%
Min	4,400 TL	$7	0.00%	0.00%	0.00%	0.00%
Median	1,135,800 TL	$4,300	0.81%	0.00%	0.38%	2.92%

Initial volume is a total annual volume given in 1994 prices and USDs. Filter 1 represents percentage in volume of all securities in total volume except for zero coupon bonds. Filter 2 gives the percentage in volume of zero coupon bonds subject to withholding tax and trading with yield less than minimum overnight rates during the month. Filter 3 shows percentage in volume of remaining bonds with less than 10 days to maturity. Note that filters were applied in consecutive order so percentages represent the exclusion after the previous filter applied, and that the statistics given at the bottom of the table are for monthly figures.

Table 2: Percentage Volumes of Most Liquid Bills (Annual Averages)

Year	Number of bills exchanged in secondary market	Volume of the most liquid bill	Volume of two most liquid bills	Volume of three most liquid bills	Herfindahl-Hirschmann index
1992	15	36%	58%	72%	22.77
1993	20	26%	42%	54%	15.02
1994	31	21%	36%	47%	10.89
1995	32	20%	35%	44%	10.47
1996	25	29%	47%	61%	17.13
1997	18	42%	68%	80%	28.96
1998	15	48%	76%	88%	36.28
1999	12	71%	83%	91%	58.60
2000	9	59%	80%	88%	43.35
2001	16	49%	66%	76%	30.73
2002	15	43%	62%	71%	26.37
2003	18	47%	64%	74%	28.05
2004	21	45%	23%	8%	27.90
Average	18.93	41%	59%	69%	27.39
Max	48	93%	98%	99%	86.95
Min	6	12%	15%	3%	6.41
Median	17	39%	61%	71%	24.12

Error Statistics

Let ε_i denote the gap between actual and fitted yields for the i^{th} bill in the sample, $r - \hat{r}$, then these error measures are given by:$RMSYE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\varepsilon_i{}^2}$, $MAYE = \frac{1}{N}\sum_{i=1}^{N}|\varepsilon_i|$, $WMAYE = \frac{1}{N}\sum_{i=1}^{N} w_i|\varepsilon_i|$, where $w_i = D(m_i)/\sum_{i=1}^{N} D(m_i)$ and $D(m_i)$ is the duration which is equal to days to maturity in zero coupon bonds.

Table 3: Annual Averages of in-sample Errors (McCulloch)

Year	MAYE	WMAYE	RMSYE	0-90	90-180	180-270	270-
1992	0.0057	0.0048	0.0076	0.0101	0.0051	0.0076	0.0045
1993	0.0051	0.0041	0.0074	0.0095	0.0051	0.0039	0.0043
1994	0.0314	0.0261	0.0446	0.0544	0.0329	0.0290	0.0424
1995	0.0118	0.0104	0.0165	0.0199	0.0139	0.0088	0.0099
1996	0.0085	0.0086	0.0114	0.0105	0.0123	0.0051	0.0028
1997	0.0079	0.0071	0.0107	0.0159	0.0089	0.0078	0.0081
1998	0.0149	0.0133	0.0200	0.0259	0.0141	0.0132	0.0137
1999	0.0137	0.0111	0.0194	0.0225	0.0141	0.0085	0.0100
2000	0.0113	0.0082	0.0155	0.0203	0.0146	0.0115	0.0078
2001	0.0165	0.0130	0.0223	0.0279	0.0130	0.0138	0.0217
2002	0.0074	0.0056	0.0100	0.0121	0.0052	0.0058	0.0056
2003	0.0048	0.0044	0.0061	0.0067	0.0055	0.0054	0.0043
2004	0.0020	0.0019	0.0026	0.0033	0.0014	0.0023	0.0021
Average	0.0114	0.0095	0.0157	0.0193	0.0118	0.0094	0.0096

Table 4: Annual Averages of in-sample Errors (Nelson-Siegel)

Year	MAYE	WMAYE	RMSYE	0-90	90-180	180-270	270-
1992	0.0063	0.0054	0.0082	0.0089	0.0069	0.0085	0.0045
1993	0.0088	0.0049	0.0147	0.0252	0.0061	0.0044	0.0046
1994	0.0414	0.0302	0.0570	0.0732	0.0372	0.0301	0.0458
1995	0.0180	0.0126	0.0263	0.0346	0.0170	0.0084	0.0108
1996	0.0113	0.0097	0.0155	0.0174	0.0132	0.0066	0.0011
1997	0.0114	0.0082	0.0188	0.0400	0.0093	0.0083	0.0090
1998	0.0159	0.0133	0.0222	0.0298	0.0159	0.0132	0.0116
1999	0.0147	0.0091	0.0236	0.0331	0.0148	0.0073	0.0072
2000	0.0138	0.0089	0.0196	0.0242	0.0164	0.0127	0.0077
2001	0.0179	0.0126	0.0272	0.0367	0.0146	0.0138	0.0212
2002	0.0065	0.0051	0.0091	0.0111	0.0046	0.0064	0.0047
2003	0.0079	0.0051	0.0118	0.0163	0.0057	0.0055	0.0048
2004	0.0021	0.0019	0.0029	0.0034	0.0021	0.0019	0.0024
Average	0.0142	0.0102	0.0208	0.0287	0.0132	0.0098	0.0094

Table 5: In-sample Analysis

	MAYE	WMAYE	RMSYE	0-90	90-180	180-270	270-
McCulloch	102	91	101	101	101	77	59
Nelson-Siegel	45	56	46	46	43	57	56

Table shows number of months in which one method outperforms another with in-sample fit according to different criteria (MAYE, WMAYE, RMSYE) as well as between specific maturities according to RMSYE.

Table 6: Out-of-sample Analysis

	ALL	0-90	90-180	180-270	270-
McCulloch	61	68	57	49	43
Nelson-Siegel	86	79	87	85	72

Table shows number of months in which one method outperforms another with out-of-sample prediction power according to RMSYE criterion for all maturities as well as between specific maturities.

Table 7: Out-of-Sample Results for RMSYE (McCulloch)

Year	All	0-90	90-180	180-270	270-
1992	0.0103	0.0096	0.0092	0.0100	0.0119
1993	0.0103	0.0083	0.0095	0.0128	0.0051
1994	0.0546	0.0396	0.0659	0.0344	0.0573
1995	0.0201	0.0156	0.0209	0.0163	0.0156
1996	0.0167	0.0135	0.0127	0.0114	0.0650
1997	0.0139	0.0124	0.0136	0.0103	0.0132
1998	0.0287	0.0233	0.0236	0.0200	0.0344
1999	0.0288	0.0222	0.0221	0.0188	0.0379
2000	0.0225	0.0291	0.0181	0.0190	0.0187
2001	0.0278	0.0293	0.0206	0.0195	0.0481
2002	0.0158	0.0132	0.0157	0.0193	0.0133
2003	0.0083	0.0077	0.0097	0.0071	0.0073
2004	0.0034	0.0033	0.0032	0.0040	0.0028
Average	0.0211	0.0183	0.0198	0.0158	0.0214

Table 8: Out-of-Sample Results for RMSYE (Nelson-Siegel)

Year	All	0-90	90-180	180-270	270-
1992	0.0083	0.0094	0.0064	0.0066	0.0083
1993	0.0194	0.0169	0.0112	0.0160	0.0194
1994	0.0653	0.0564	0.0625	0.0357	0.0531
1995	0.0245	0.0215	0.0253	0.0228	0.0281
1996	0.0162	0.0153	0.0159	0.0134	0.0041
1997	0.0192	0.0296	0.0145	0.0097	0.0127
1998	0.0207	0.0184	0.0208	0.0182	0.0185
1999	0.0262	0.0213	0.0178	0.0161	0.0289
2000	0.0186	0.0259	0.0093	0.0076	0.0136
2001	0.0309	0.0337	0.0199	0.0113	0.0619
2002	0.0087	0.0082	0.0081	0.0071	0.0035
2003	0.0136	0.0136	0.0164	0.0064	0.0070
2004	0.0030	0.0033	0.0027	0.0018	0.0026
Average	0.0222	0.0221	0.0189	0.0135	0.0185

Figures

Figure 1: Monthly Volume averages

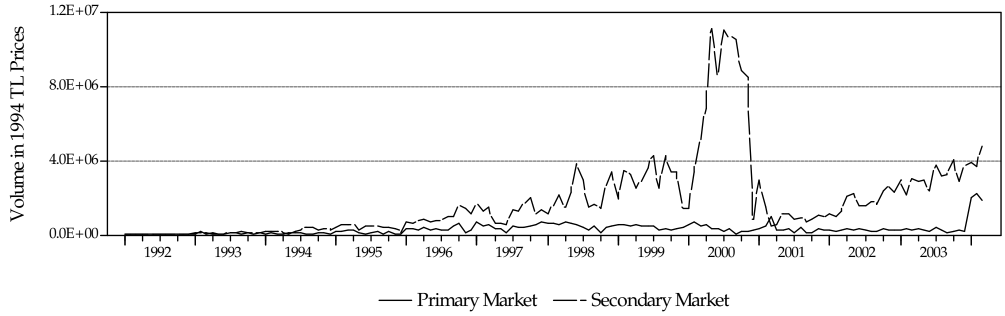


Figure 2: Monthly Maturity Averages

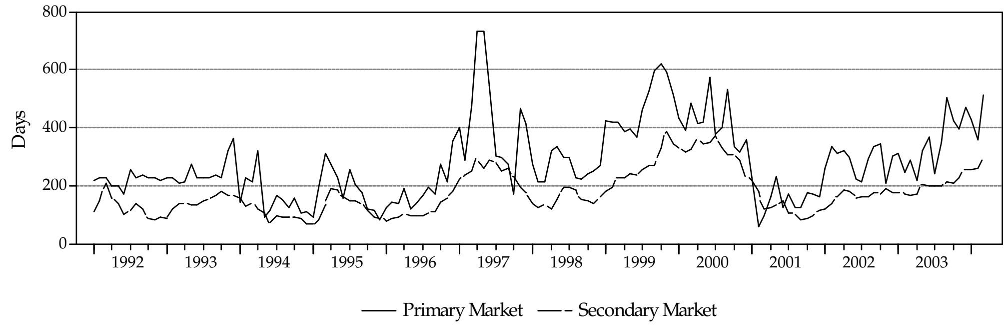


Figure 3: Monthly Continuously Compounded Yield Averages

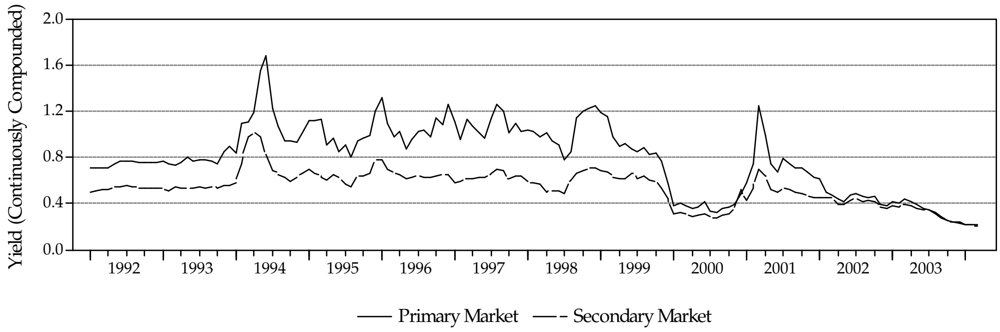
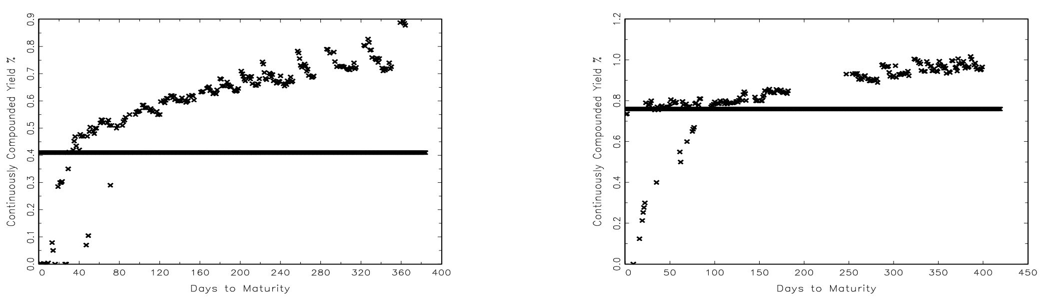


Figure 4: Yield Distortions Caused By Withholding Tax



Graphs show examples of obvious distortions on yield caused by Withholding Tax during July 1998 and May 1999. Solid line represents the minimum overnight interest rates during the month.

Figure 5: Examples of Fitted Yield Curves

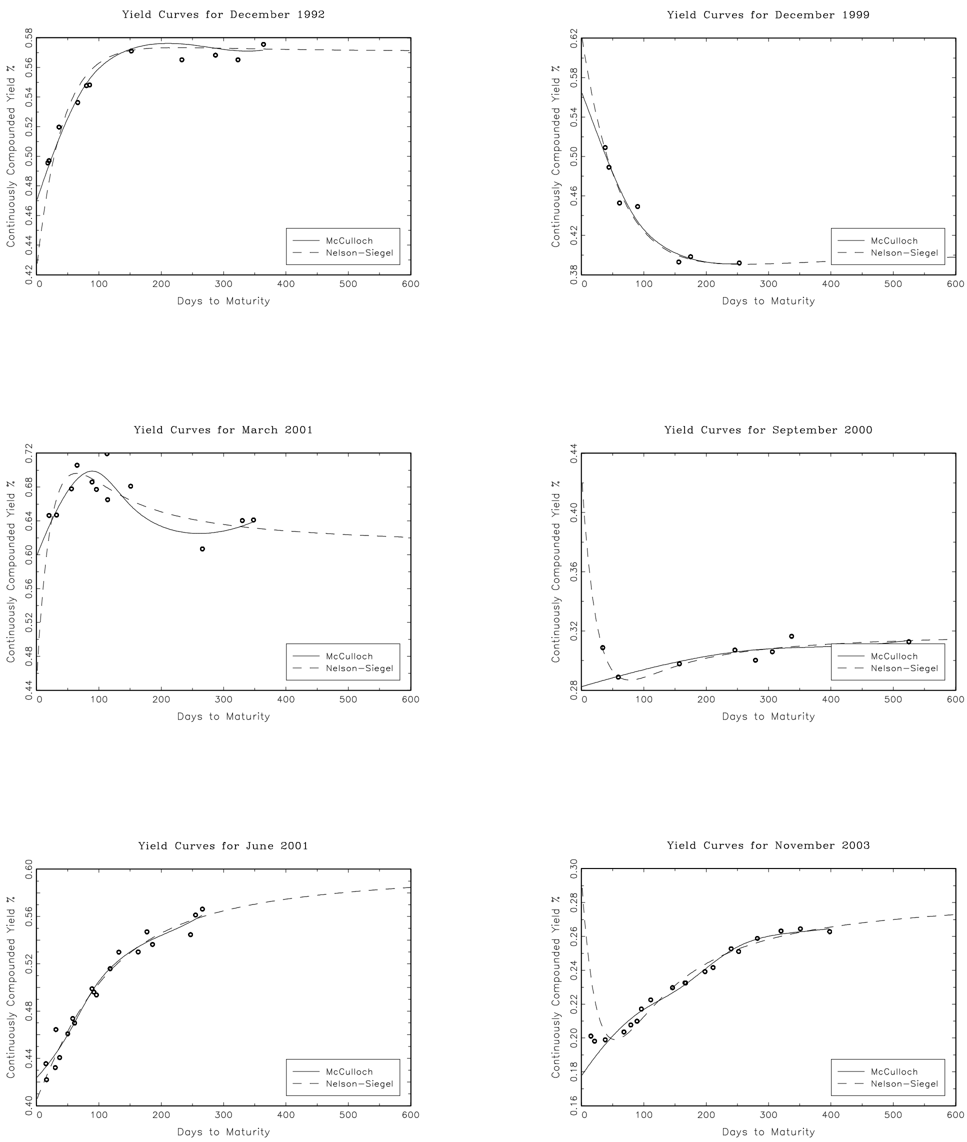


Figure 6: Estimated Term Premia Using Both Methods(Estimated with $\eta(365)-\eta(30)$)

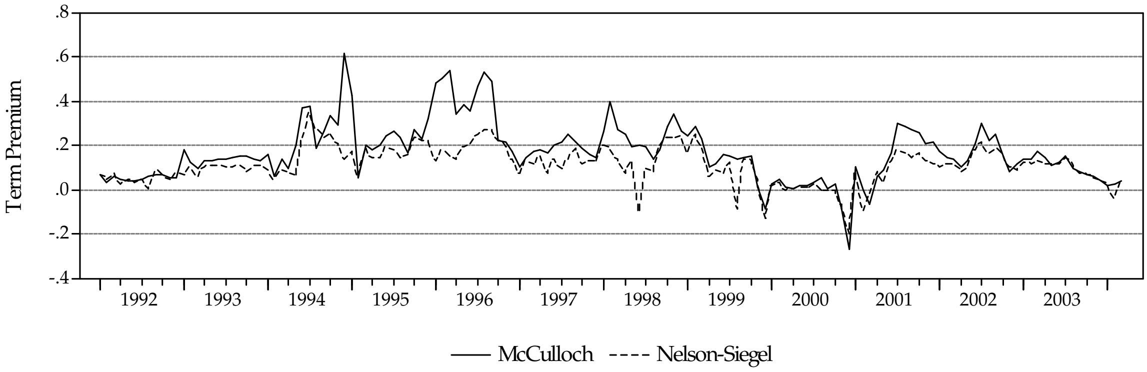
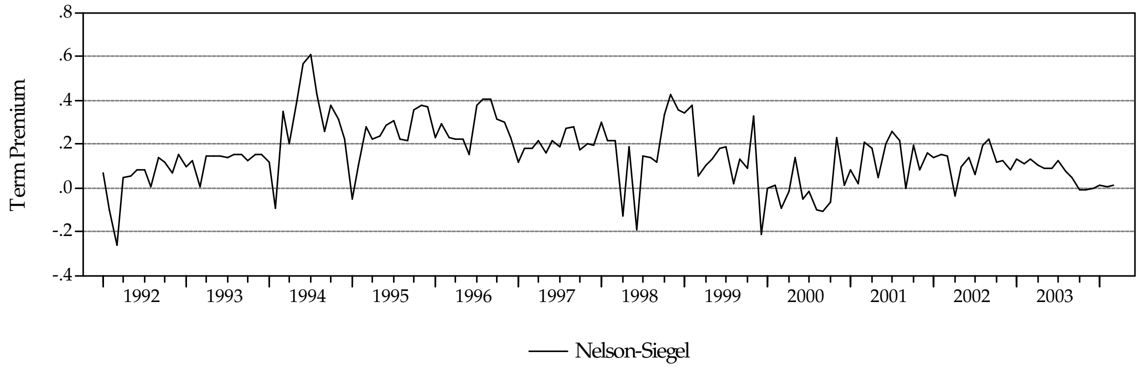


Figure 7: Estimated Term Premia Using Nelson-Siegel Model (Estimated with $-\beta_1$)

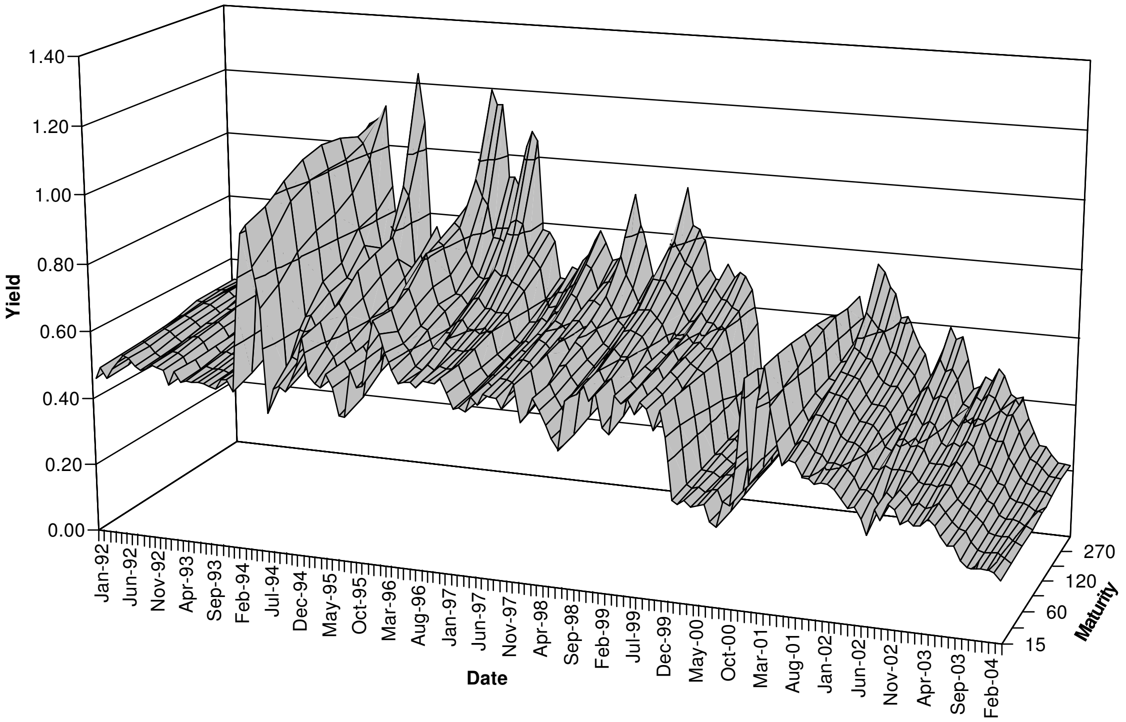


We estimate the term premium by $\eta(365) - \eta(30)$ for both methods and as $-\beta_1$ for Nelson-Siegel model. Remarkable differences between estimated term premiums can be explained by low out-of-sample prediction ability of McCulloch in the long end of the yield curve. The common characteristic of the months when there is big difference between estimated term premiums is that the sample does not include observations with one year maturity or higher. As a result, in this months McCulloch method tends to overestimate one year yield, thus overestimating term premium.

Estimation Results

Figure 8: Yield Curves January 1992 - March 2004

McCulloch



Nelson-Siegel

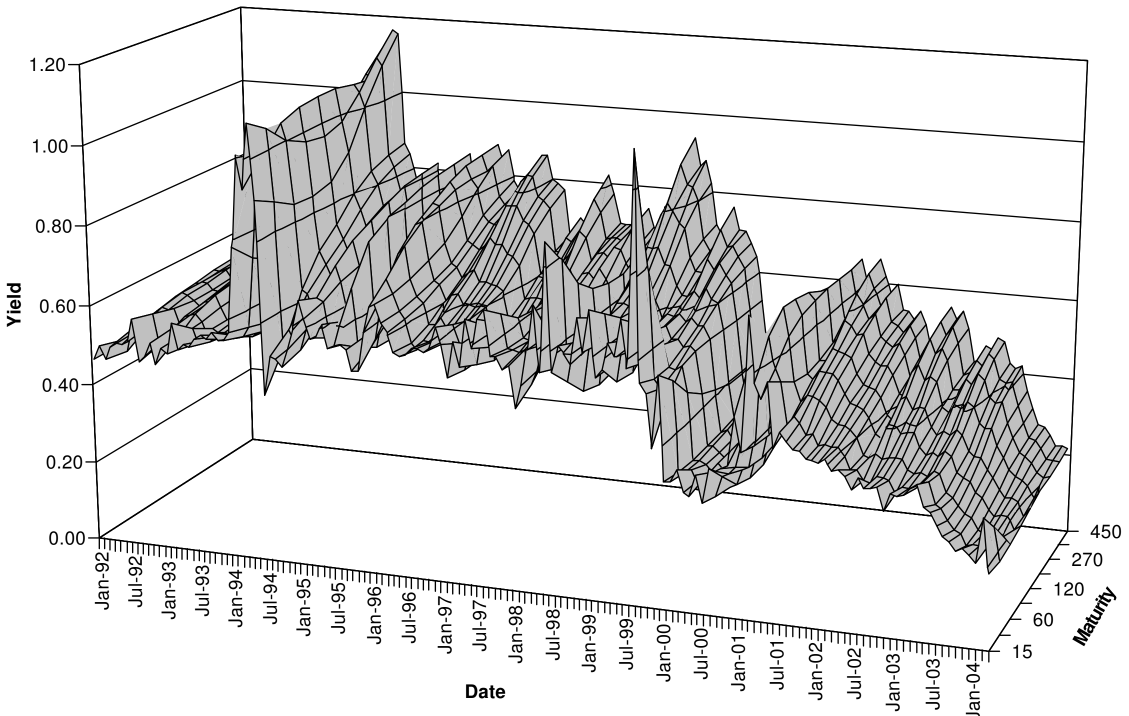


Table 11: Estimated McCulloch continuously compounded yield curve series and Nelson-Siegel model parameters for monthly average data

		McCulloch yield curve series											Nelson-Siegel parameters				
1992		15	30	60	90	120	180	270	365	Term P.*	1992		β_0	β_1	β_2	τ	Term P.†
	1	46.05	46.84	48.36	49.76	51.01	52.75	53.26	‡**53.45**	6.61		1	54.63	-6.50	-15.80	17.94	6.50
	2	49.52	49.88	50.57	51.20	51.76	52.65	53.20	**53.24**	3.36		2	53.93	10.41	-32.49	10.36	-10.41
	3	46.29	47.07	48.58	49.97	51.20	52.87	53.31	**53.26**	6.19		3	54.30	26.54	-65.22	8.05	-26.54
	4	49.59	50.07	50.99	51.81	52.53	53.49	53.78	**54.31**	4.24		4	53.96	-4.79	-8.03	6.10	4.79
	5	51.61	51.94	52.58	53.18	53.75	54.78	55.58	**55.70**	3.76		5	56.68	-5.07	-8.90	23.65	5.07
	6	53.78	54.20	54.96	55.58	56.05	56.36	56.02	**57.93**	3.73		6	55.50	-7.83	11.31	37.26	7.83
	7	52.96	53.45	54.37	55.19	55.90	56.86	57.16	**57.72**	4.27		7	58.47	-8.09	-0.01	38.96	8.09
	8	50.83	51.57	52.95	54.11	54.98	55.46	55.12	**57.45**	5.88		8	59.99	-0.43	-18.96	59.56	0.43
	9	49.81	50.36	51.46	52.53	53.54	55.36	56.74	**56.97**	6.61		9	58.72	-13.34	-5.68	31.58	13.34
	10	50.40	51.21	52.71	54.00	55.01	55.95	56.22	**57.56**	6.35		10	57.56	-11.80	-1.62	22.91	11.80
	11	51.73	52.31	53.38	54.29	54.99	55.68	56.09	**57.27**	4.96		11	58.22	-6.54	-3.10	51.67	6.54
	12	51.30	52.08	53.57	54.92	56.09	57.58	57.52	**57.05**	4.97		12	56.97	-14.86	17.30	42.11	14.86
1993		15	30	60	90	120	180	270	365	Term P.	1993		β_0	β_1	β_2	τ	Term P.
	1	50.47	51.13	52.30	53.18	53.81	55.35	**60.19**	**68.78**	17.65		1	60.74	-9.75	-10.22	50.99	9.75
	2	46.36	47.20	48.84	50.41	51.86	54.21	56.48	**59.29**	12.09		2	64.25	-12.41	-29.89	47.22	12.41
	3	50.50	50.67	51.10	51.68	52.45	54.77	58.05	**59.92**	9.25		3	65.08	-0.31	-48.63	39.24	0.31
	4	47.46	48.11	49.44	50.80	52.18	54.95	58.02	**60.83**	12.72		4	67.29	-14.11	-33.22	53.18	14.11
	5	48.37	48.86	49.90	51.04	52.31	55.19	58.50	**62.19**	13.33		5	69.93	-14.64	-40.85	55.29	14.64
	6	47.98	48.38	49.32	50.47	51.89	54.96	58.19	**61.72**	13.34		6	69.36	-14.42	-40.46	54.94	14.42
	7	47.97	48.52	49.69	50.95	52.33	55.19	58.89	**62.52**	14.00		7	71.78	-13.95	-47.23	57.82	13.95
	8	47.89	48.32	49.25	50.29	51.48	54.39	59.02	**62.98**	14.66		8	74.37	-14.74	-57.59	59.64	14.74
	9	47.39	47.93	49.06	50.27	51.58	54.56	58.87	**62.98**	15.05		9	73.75	-15.09	-55.31	58.65	15.09
	10	46.65	47.18	48.26	49.39	50.56	53.07	57.04	**61.98**	14.80		10	73.66	-12.24	-62.66	61.42	12.24
	11	49.65	49.97	50.73	51.70	52.94	55.91	59.83	**63.33**	13.36		11	72.47	-14.81	-45.98	57.66	14.81
	12	49.98	50.36	51.20	52.18	53.32	56.09	60.29	**63.53**	13.17		12	72.80	-14.92	-45.78	57.80	14.92
1994		15	30	60	90	120	180	270	365	Term P.	1994		β_0	β_1	β_2	τ	Term P.
	1	46.58	48.47	51.91	54.78	56.88	58.17	59.69	**64.37**	15.90		1	71.19	-11.75	-37.15	59.44	11.75
	2	71.50	71.54	71.60	71.74	72.16	74.36	76.84	**77.26**	5.72		2	80.28	9.52	-47.88	26.50	-9.52
	3	92.75	92.74	92.46	92.23	92.69	96.01	100.80	**106.28**	13.54		3	138.79	-34.65	-91.24	104.14	34.65
	4	95.47	97.89	102.39	105.99	107.96	107.24	104.94	**107.60**	9.71		4	103.80	-20.17	35.46	37.93	20.17
	5	83.73	88.95	96.67	99.93	100.88	100.89	103.02	**108.99**	20.04		5	153.29	-39.95	-118.12	113.34	39.95
	6	71.69	75.70	82.06	85.33	87.15	90.64	98.23	**112.38**	36.68		6	150.63	-56.31	-117.47	94.32	56.31
	7	41.33	50.07	63.03	69.99	72.59	80.38	**82.14**	**87.35**	37.28		7	91.04	-60.45	-10.19	28.71	60.45
	8	48.64	54.66	63.37	66.98	68.42	**74.89**	**71.41**	**73.49**	18.83		8	89.70	-42.17	-13.06	47.50	42.17
	9	49.63	53.33	59.28	62.65	67.55	70.76	**72.24**	**79.34**	26.01		9	80.73	-25.40	-49.86	21.54	25.40
	10	48.06	50.62	55.17	58.51	61.82	67.75	**72.80**	**83.78**	33.16		10	86.13	-37.74	-34.33	48.40	37.74

Table 9: (cont.)

		McCulloch yield curve series								
	11	51.60	55.49	60.94	63.07	65.86	**70.03**	**74.34**	**84.78**	29.29
	12	55.74	61.62	63.72	69.62	67.67	**73.44**	**90.94**	**122.95**	61.33
1995		15	30	60	90	120	180	270	365	Term P.
	1	64.42	65.74	67.79	74.33	76.35	**78.48**	**88.25**	**108.22**	42.48
	2	53.74	58.95	64.63	66.42	69.60	71.85	**65.63**	**64.06**	5.11
	3	51.50	55.65	56.87	59.69	62.57	66.26	70.27	**75.87**	20.22
	4	50.55	51.36	52.85	54.13	55.22	58.27	64.44	**69.41**	18.05
	5	54.52	54.91	56.03	57.70	60.12	65.31	68.24	**75.17**	20.26
	6	50.88	51.24	52.52	54.70	57.69	62.42	67.79	**75.23**	23.99
	7	42.92	45.12	48.95	51.88	54.05	58.64	66.27	**71.14**	26.02
	8	42.47	44.92	48.93	51.52	52.75	55.92	62.27	**68.17**	23.25
	9	56.58	56.56	57.33	59.52	62.59	66.31	68.91	**73.03**	16.47
	10	51.61	53.32	57.00	61.10	65.29	69.36	73.29	**80.28**	26.96
	11	52.06	56.45	63.10	66.34	69.42	74.58	75.92	**79.60**	23.15
	12	62.76	66.73	73.73	79.03	81.99	84.60	**88.05**	**98.43**	31.70
1996		15	30	60	90	120	180	270	365	Term P.
	1	71.50	71.21	74.25	81.34	80.62	83.15	**94.96**	**119.58**	48.37
	2	67.66	65.35	68.94	71.72	72.20	76.37	**89.82**	**115.34**	49.99
	3	62.47	61.74	63.55	66.69	66.75	**71.77**	**87.43**	**115.42**	53.68
	4	57.61	59.43	62.60	64.89	66.15	68.98	**77.47**	**93.26**	33.83
	5	53.97	55.41	57.58	59.96	63.71	68.18	**77.18**	**93.53**	38.12
	6	54.66	54.62	56.88	61.74	65.47	68.94	**76.16**	**90.18**	35.56
	7	54.71	56.18	59.33	62.76	66.01	71.43	**82.67**	**103.14**	46.96
	8	53.30	54.78	57.67	60.41	63.10	69.52	**83.47**	**107.80**	53.02
	9	55.57	56.13	57.67	59.93	63.10	69.93	**82.60**	**104.96**	48.83
	10	55.42	55.85	57.14	59.18	62.18	68.11	73.11	**77.92**	22.07
	11	54.99	55.90	57.85	60.00	62.38	67.25	72.13	**77.57**	21.67
	12	56.80	57.34	58.57	60.06	61.87	65.45	69.58	74.43	17.09
1997		15	30	60	90	120	180	270	365	Term P.
	1	52.86	52.96	53.25	53.72	54.44	56.59	59.88	63.42	10.46
	2	48.28	48.97	50.38	51.87	53.43	56.63	60.75	63.43	14.46
	3	49.09	49.86	51.42	53.02	54.65	57.94	62.66	67.24	17.38
	4	47.96	48.68	50.16	51.71	53.34	56.84	62.50	66.39	17.71
	5	52.52	53.10	54.30	55.53	56.80	59.46	63.88	**69.79**	16.69
	6	50.45	51.32	53.11	54.94	56.79	60.48	65.54	**71.32**	20.00
	7	53.55	54.41	56.13	57.83	59.48	62.47	66.78	**76.14**	21.73
	8	53.02	54.20	56.60	59.01	61.42	65.97	71.82	**79.26**	25.06

		Nelson-Siegel parameters				
	11	83.27	-31.45	-19.32	46.82	31.45
	12	77.88	-21.95	0.07	50.49	21.95
1995		β_0	β_1	β_2	τ	Term P.
	1	83.68	5.59	-74.00	13.21	-5.59
	2	70.56	-9.90	-6.76	18.88	9.90
	3	86.93	-28.01	-45.87	58.92	28.01
	4	84.14	-21.94	-67.49	62.19	21.94
	5	87.46	-23.33	-57.19	64.13	23.33
	6	87.12	-28.48	-60.92	58.64	28.48
	7	86.09	-30.41	-69.43	5.67	30.41
	8	83.98	-21.92	-76.26	62.06	21.92
	9	77.54	-21.71	-27.10	44.68	21.71
	10	83.25	-35.43	-19.06	47.82	35.43
	11	81.66	-37.28	4.25	44.38	37.28
	12	89.41	-36.87	23.87	52.55	36.87
1996		β_0	β_1	β_2	τ	Term P.
	1	83.10	-22.57	25.47	54.00	22.57
	2	96.86	-29.04	-37.13	67.82	29.04
	3	86.08	-22.53	-29.86	63.54	22.53
	4	82.44	-21.82	-22.90	60.62	21.82
	5	80.91	-21.89	-44.26	35.49	21.89
	6	77.80	-15.39	-53.40	22.64	15.39
	7	92.86	-37.42	-39.50	55.44	37.42
	8	100.30	-40.69	-66.50	59.61	40.69
	9	100.23	-40.54	-65.77	59.69	40.54
	10	89.64	-30.94	-51.73	52.61	30.94
	11	89.39	-29.81	-48.18	59.58	29.81
	12	89.17	-22.75	-58.32	66.42	22.75
1997		β_0	β_1	β_2	τ	Term P.
	1	74.45	-11.25	-52.16	63.21	11.25
	2	71.52	-18.21	-37.75	53.31	18.21
	3	83.37	-18.16	-73.09	65.21	18.16
	4	79.39	-21.26	-60.92	58.13	21.26
	5	87.29	-15.52	-80.01	71.77	15.52
	6	84.10	-21.29	-61.69	62.81	21.29
	7	89.45	-18.53	-73.05	70.92	18.53
	8	101.28	-27.35	-89.52	73.93	27.35

Table 9: (cont.)

		McCulloch yield curve series								
	9	52.69	53.76	55.95	58.20	60.50	65.11	70.80	**75.44**	21.68
	10	49.99	51.40	54.08	56.52	58.61	61.25	64.44	**70.22**	18.82
	11	53.76	55.06	57.56	59.86	61.86	64.53	66.81	**70.94**	15.88
	12	57.05	57.69	59.03	60.44	61.93	64.93	67.24	**71.78**	14.09
1998		15	30	60	90	120	180	270	365	Term P.
	1	46.48	48.67	52.77	56.44	59.50	63.19	**67.08**	**75.12**	26.45
	2	49.12	51.44	55.75	59.51	62.49	**66.39**	**75.22**	**90.94**	39.50
	3	53.81	54.68	56.43	58.20	59.94	63.29	**69.95**	**81.94**	27.26
	4	50.62	51.39	52.96	54.54	56.14	59.43	65.75	**76.28**	24.89
	5	46.47	47.78	50.31	52.68	54.83	58.11	61.86	**66.79**	19.01
	6	42.92	43.94	45.98	48.03	50.08	54.03	59.00	**63.95**	20.01
	7	39.78	40.53	42.07	43.65	45.26	48.58	53.64	**59.95**	19.42
	8	54.10	54.79	56.15	57.50	58.80	61.19	64.28	**68.76**	13.97
	9	55.57	56.65	58.92	61.37	64.01	69.83	73.80	**74.34**	17.69
	10	56.18	57.67	60.72	63.83	66.95	72.78	78.10	**85.96**	28.29
	11	59.11	60.13	62.30	64.64	67.17	72.71	81.07	**94.32**	34.19
	12	54.72	56.63	60.40	64.05	67.46	72.89	78.21	83.13	26.50
1999		15	30	60	90	120	180	270	365	Term P.
	1	56.81	58.02	60.44	62.84	65.17	69.33	74.98	82.03	24.01
	2	47.55	50.17	55.10	59.53	63.27	68.47	73.72	78.87	28.70
	3	45.85	47.64	51.17	54.64	57.98	63.76	68.17	70.20	22.56
	4	57.16	57.54	58.28	59.00	59.71	61.16	63.63	67.38	9.84
	5	54.85	55.52	56.84	58.12	59.34	61.51	64.08	67.20	11.68
	6	56.41	57.09	58.47	59.86	61.25	64.02	68.05	72.53	15.44
	7	52.49	53.34	55.05	56.76	58.44	61.58	65.20	68.59	15.25
	8	55.94	56.41	57.34	58.28	59.23	61.23	64.80	70.29	13.88
	9	54.30	54.73	55.58	56.46	57.36	59.34	63.11	**69.30**	14.57
	10	48.52	49.95	52.68	55.20	57.39	60.26	62.05	**65.13**	15.18
	11	54.39	54.36	54.22	53.99	53.71	53.14	53.00	**55.15**	0.79
	12	46.98	46.45	45.32	44.10	42.84	40.43	**38.02**	**37.62**	-8.83
2000		15	30	60	90	120	180	270	365	Term P.
	1	29.35	29.64	30.16	30.58	30.91	31.25	31.48	31.75	2.11
	2	28.16	28.82	29.99	30.92	31.57	32.24	32.72	33.16	4.34
	3	29.51	29.53	29.58	29.62	29.67	29.79	30.09	30.69	1.16
	4	27.96	28.07	28.26	28.42	28.53	28.63	28.66	28.77	0.70
	5	28.65	28.88	29.31	29.71	30.07	30.63	30.93	30.49	1.61
	6	29.24	29.29	29.38	29.47	29.57	29.81	30.28	31.07	1.78

		Nelson-Siegel parameters				
	9	87.97	-28.05	-52.73	59.51	28.05
	10	77.33	-17.52	-40.32	59.81	17.52
	11	73.29	-19.93	-12.50	53.36	19.93
	12	73.45	-18.96	-12.24	54.48	18.96
1998		β_0	β_1	β_2	τ	Term P.
	1	72.58	-30.15	-15.10	38.16	30.15
	2	74.59	-21.17	-37.10	25.91	21.17
	3	84.12	-21.63	-49.97	62.49	21.63
	4	75.93	13.18	-96.48	34.85	-13.18
	5	76.51	-18.79	-48.84	57.72	18.79
	6	76.00	19.42	-62.96	56.97	-19.42
	7	76.83	-14.40	-89.26	62.42	14.40
	8	78.29	-13.41	-46.58	64.87	13.41
	9	81.18	-11.26	-65.29	27.04	11.26
	10	91.45	-33.14	-42.89	47.73	33.14
	11	116.22	-42.76	-96.85	73.46	42.76
	12	97.95	-35.52	-55.94	56.49	35.52
1999		β_0	β_1	β_2	τ	Term P.
	1	110.04	-33.84	-98.13	76.20	33.84
	2	93.83	-37.48	-54.10	56.36	37.48
	3	77.10	-5.51	-83.53	27.47	5.51
	4	76.79	-10.28	-42.87	66.51	10.28
	5	75.78	-13.08	-37.80	62.52	13.08
	6	93.06	-17.95	-82.64	75.11	17.95
	7	79.21	-18.83	-45.13	60.38	18.83
	8	74.02	-6.15	-41.57	23.71	6.15
	9	72.36	-12.94	-44.89	36.82	12.94
	10	66.67	-8.68	-39.32	24.58	8.68
	11	52.27	-32.49	42.20	19.79	32.49
	12	40.90	21.67	-31.08	62.57	-21.67
2000		β_0	β_1	β_2	τ	Term P.
	1	33.00	0.07	-9.96	33.07	-0.07
	2	34.89	-1.27	-13.52	33.62	1.27
	3	31.87	9.47	-22.13	26.89	-9.47
	4	29.47	1.60	-7.30	31.07	-1.60
	5	30.35	-13.75	-4.55	1.34	13.75
	6	33.39	5.35	-25.59	38.73	-5.35

Table 9: (cont.)

McCulloch yield curve series										
	7	24.40	24.61	25.04	25.45	25.84	26.58	27.47	28.08	3.47
	8	23.12	23.42	24.01	24.58	25.14	26.21	27.60	28.71	5.29
	9	30.35	30.37	30.41	30.43	30.45	30.48	30.53	30.69	0.32
	10	29.43	29.55	29.79	30.02	30.25	30.69	31.34	32.09	2.54
	11	43.06	42.29	40.70	39.10	37.55	34.95	33.17	33.48	-8.81
	12	65.84	64.57	61.57	58.09	54.33	47.01	40.08	37.76	-26.81
2001		15	30	60	90	120	180	270	365	Term P.
	1	35.92	36.67	38.12	39.49	40.76	42.87	44.83	46.66	9.99
	2	58.00	56.00	51.95	48.19	45.18	42.94	46.88	55.52	-0.48
	3	69.63	69.49	69.02	68.39	67.71	66.39	64.24	**63.04**	-6.45
	4	58.70	60.00	62.43	64.49	65.99	66.92	65.98	**66.30**	6.30
	5	48.65	49.40	50.73	51.74	52.41	53.42	55.21	58.24	8.84
	6	42.45	44.15	47.21	49.71	51.59	53.98	**56.24**	**60.93**	16.78
	7	44.21	46.13	49.57	52.37	54.51	58.16	**64.97**	**76.23**	30.10
	8	43.65	45.61	49.12	51.96	54.05	57.19	**63.49**	**74.07**	28.46
	9	39.97	43.01	48.10	51.70	53.56	**55.48**	**60.77**	**69.90**	26.89
	10	39.65	42.47	47.22	50.63	52.67	55.30	**59.90**	**68.00**	25.53
	11	38.23	40.49	44.33	47.13	48.76	50.64	**54.33**	**60.89**	20.40
	12	39.05	40.47	42.95	44.85	46.23	48.75	**53.82**	**61.62**	21.15
2002		15	30	60	90	120	180	270	365	Term P.
	1	38.73	40.23	42.81	44.71	45.94	47.81	**51.50**	**57.66**	17.43
	2	37.43	39.00	41.81	44.11	45.79	47.83	50.07	**53.10**	14.10
	3	35.05	36.86	39.99	42.40	43.99	45.80	47.81	**50.87**	14.01
	4	32.59	33.84	36.13	38.10	39.69	41.52	42.66	**44.00**	10.16
	5	33.23	34.06	35.67	37.17	38.55	40.80	43.38	**46.74**	12.68
	6	30.26	32.66	36.98	40.67	43.64	47.07	49.62	**53.20**	20.54
	7	25.68	29.47	35.98	41.15	44.87	48.98	53.29	**59.62**	30.15
	8	33.05	34.09	36.33	38.81	41.37	45.29	**49.74**	**55.97**	21.88
	9	30.18	32.34	36.27	39.71	42.58	46.49	50.95	**57.00**	24.66
	10	35.02	35.10	36.18	38.65	41.86	45.83	47.94	**50.51**	15.41
	11	33.80	33.40	33.48	34.88	37.00	39.76	40.94	41.58	8.18
	12	29.73	30.89	32.99	34.74	36.08	37.85	40.02	**42.81**	11.92
2003		15	30	60	90	120	180	270	365	Term P.
	1	31.36	31.87	33.23	35.08	37.20	40.46	43.16	45.78	13.91
	2	29.75	30.70	32.58	34.42	36.19	39.25	42.08	44.51	13.81
	3	30.14	31.37	33.73	35.96	38.02	41.12	44.49	**48.40**	17.03
	4	32.24	32.51	33.40	34.82	36.70	40.23	43.54	46.62	14.11

Nelson-Siegel parameters						
	7	29.63	1.51	-19.59	31.14	-1.51
	8	33.74	10.28	-48.95	44.03	-10.28
	9	32.08	10.68	-25.29	26.33	-10.68
	10	35.16	6.52	-29.86	41.68	-6.52
	11	33.21	-22.75	54.67	10.46	22.75
	12	29.96	-1.38	107.56	28.58	1.38
2001		β_0	β_1	β_2	τ	Term P.
	1	51.83	-8.03	-31.99	43.80	8.03
	2	86.23	-1.79	-142.02	84.44	1.79
	3	60.68	-20.80	58.51	23.67	20.80
	4	65.08	-17.71	24.50	41.36	17.71
	5	66.61	-4.38	-45.43	62.23	4.38
	6	60.49	-19.87	-9.66	40.62	19.87
	7	71.98	-25.49	-29.60	46.48	25.49
	8	65.25	-21.66	-20.40	34.42	21.66
	9	59.13	0.33	-58.35	11.79	-0.33
	10	62.88	-19.35	-29.38	26.11	19.35
	11	55.33	-8.14	-36.75	17.25	8.14
	12	56.31	-15.71	-18.07	40.60	15.71
2002		β_0	β_1	β_2	τ	Term P.
	1	53.74	-13.42	-16.29	34.76	13.42
	2	57.57	-15.02	-26.73	42.55	15.02
	3	53.34	-14.22	-20.49	39.11	14.22
	4	45.76	4.06	-41.78	20.37	-4.06
	5	49.92	-9.29	-32.57	40.63	9.29
	6	54.93	-13.38	-48.60	22.75	13.38
	7	59.75	-5.62	-81.71	21.41	5.62
	8	57.37	-19.58	-38.89	37.79	19.58
	9	60.22	-21.97	-43.53	38.25	21.97
	10	53.50	-11.60	-44.59	25.96	11.60
	11	43.53	-12.06	-19.79	23.86	12.06
	12	46.19	-8.30	-31.33	37.89	8.30
2003		β_0	β_1	β_2	τ	Term P.
	1	50.64	-12.76	-37.70	37.89	12.76
	2	49.52	-10.90	-38.82	38.62	10.90
	3	54.13	-13.15	-45.97	40.98	13.15
	4	53.25	-10.40	-48.97	42.85	10.40

Table 9: (cont.)

		McCulloch yield curve series											Nelson-Siegel parameters				
	5	30.67	30.12	29.67	30.22	31.75	35.65	40.08	41.29	11.17		5	46.72	-8.62	-45.75	36.02	8.62
	6	27.78	27.90	28.51	29.66	31.30	34.78	38.51	40.22	12.32		6	45.70	-8.95	-45.16	36.75	8.95
	7	25.47	25.98	27.23	28.80	30.73	34.91	39.05	40.73	14.75		7	46.51	-12.36	-48.60	34.15	12.36
	8	25.62	25.39	25.33	25.90	27.15	30.58	34.25	34.62	9.23		8	38.48	-7.56	-36.82	29.94	7.56
	9	21.93	22.59	23.87	25.09	26.24	28.25	29.92	30.66	8.07		9	33.05	-4.47	-25.66	28.58	4.47
	10	20.14	20.53	21.36	22.24	23.20	25.12	26.63	26.90	6.37		10	28.82	0.91	-30.59	21.84	-0.91
	11	19.89	20.11	20.63	21.24	21.95	23.68	25.63	26.33	6.22		11	28.78	0.81	-30.79	29.59	-0.81
	12	20.18	20.36	20.76	21.24	21.80	23.08	24.16	24.45	4.09		12	25.83	0.41	-18.95	26.24	-0.41
2004		15	30	60	90	120	180	270	365	Term P.	2004		β_0	β_1	β_2	τ	Term P.
	1	20.20	20.38	20.71	21.01	21.28	21.68	21.91	22.05	1.67		1	22.49	-0.87	-5.95	21.62	0.87
	2	19.49	19.46	19.52	19.76	20.19	21.28	21.94	21.94	2.48		2	25.21	15.59	-45.92	40.80	-15.59
	3	17.58	17.89	18.49	19.08	19.64	20.63	21.39	21.58	3.69		3	22.43	-1.25	-13.68	21.18	1.25

*Estimated as $\eta(365) - \eta(30)$

†Estimated as $-\beta_1$

‡Bold numbers indicate estimations beyond the maximum maturity observed.